

Azonosító
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ÉRETTSÉGI VIZSGA • 2020. május 5.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2020. május 5. 9:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTERIUMA

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Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.
2. You may solve the problems in any order.
3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below.** *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

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4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!
5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**
6. **Make sure that calculations of intermediate results are also possible to follow.**
7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations:** addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e , finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.
9. Always state the final result (the answer to the question of the problem) in words, too!

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10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.
11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.
12. Please, **do not write in the grey rectangles**.

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I.

1. The sides of rectangle $ABCD$ are $AB = 17$ cm and $AD = 8$ cm.

- a) Calculate the area of the part of the rectangle that is covered by a circle of centre A and radius 17 cm.
- b) Vertex P of the rhombus $APCQ$ is an interior point of side AB of the rectangle. Vertex Q is an interior point of side CD . Calculate the perimeter of the rhombus.

a)	7 points	
b)	5 points	
T.:	12 points	

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2. a) Solve the following equation in the set of real numbers.

$$|x - 2| = 7 + x - 0,25x^2$$

- b) How many integer solutions does the following inequality have?

$$\log_2(x^2 - 200) < 20$$

a)	7 points	
b)	6 points	
T.:	13 points	

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- 3.** The following two articles appeared on several news websites during the summer of 2018.
- I. In July, 2018, a total 67 000 people had been unemployed for more than one year. This was 26.0% of those looking for a job.
- II. In July, 2018, the ratio of those looking for jobs was 3.8% of the working-age population, while it was 5.6% of the active population. (We may safely assume that the active population is a subset of the working-age population.)
- a) Use the data above to calculate the number of people seeking jobs. Round your answer to the nearest ten thousand.
- b) What percentage is the active population of the working-age population?

Another news article from 2018 reported that the mean monthly gross income in December, 2017, was 328 000 Ft, while the median of the monthly gross incomes was about 256 000 Ft.

- c) Give 7 different positive numbers such that their mean is greater than their median. Also give the mean and the median of these numbers.
- d) Mr. Virág believes that he earns more than half of all working people. His salary therefore, in his opinion, should be higher than the mean income and yet, his gross income is only 283 000 Ft per month. He says that this proves the above statistics to be wrong.
Explain briefly (in 1-2 sentences) why Mr. Virág's conclusion is incorrect.

a)	3 points	
b)	4 points	
c)	3 points	
d)	2 points	
T.:	12 points	

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4. Given is the line $e: y = 7 - \frac{1}{2}x$.

a) One vertex of a square is the origin and one diagonal is line e . Calculate the coordinates of the centre of the square and also the area of the square.

b) Calculate the area of the finite planar figure that is bounded by the parabola $y = -\frac{(x-4)^2}{4} + 7$ and the line e .

a)	7 points	
b)	7 points	
T.:	14 points	

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II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. The first four terms of a strictly increasing sequence are all elements of the set $\{1; 2; 3; 4; 5; 6; 7; 8; 9\}$, such that there are no consecutive integers among the terms.

a) How many different ways are there to select the first four terms of this sequence?

One of the three-digit positive integers is randomly selected.

b) Determine the probability that the digits of the selected number will be, from left to right, consecutive digits in increasing or decreasing order.

The digits a, b, c and d are consecutive numbers in the decimal (base 10) system, $a \neq 0$ and $a < b < c < d$. The base 9 form of a certain number N is \overline{abc} , while the base 8 form of the same number is \overline{bcd} .

c) Give the number N in decimal (base 10) form.

a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

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6. The shortest diagonal of a regular decagon (10 sided figure) is 6 cm long.

a) Determine the length of the side of the decagon.

Let G be a simple graph on 10 vertices that has 6 edges.

b) Is it true that among the vertices of graph G there are at least two whose degree is at least 2? Explain your answer.

One of the edges of a complete graph on n vertices has been coloured red ($n \geq 3$). Select one of the remaining edges at random. Event A is that the selected edge and the red edge do share a common vertex. Event B is that the selected edge and the red edge do not share a common vertex.

c) How many vertices does this graph have if the probabilities of events A and B are equal?

a)	4 points	
b)	3 points	
c)	9 points	
T.:	16 points	

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7. Players in a game build four-tier pyramids out of equal-sized balls (*see diagram*). The pyramids consist of $1^2 + 2^2 + 3^2 + 4^2 = 30$ balls, 15 of which are of dark colour, the other 15 are of light colour. The 30 balls are placed in a box before the game begins and then they are randomly pulled from that box.

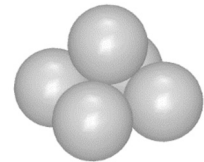


- a) Calculate the probability that all 4 balls in the second layer from the top down will be of dark colour.

A similar pyramid of n tiers consists of $1^2 + 2^2 + \dots + n^2$ balls ($n \in \mathbf{N}^+$).

- b) Prove (e.g. by induction) that $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

A 2-tier pyramid consists of 5 balls (*see diagram*). The 4 balls forming the lower tier are placed on a horizontal surface, touching each other, such that their centres form the four vertices of a square. The radius of each ball is 1 cm.



- c) Calculate the height of the 2-tier pyramid.

a)	4 points	
b)	6 points	
c)	6 points	
T.:	16 points	

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- 8.** A factory that produces food items has 700 kg of powdered soup in its warehouse. To reduce stock, the sales manager of the factory plans to first decrease the current stock by 24% (by sales, charity giveaways, etc.) each month and then add the 60 kg monthly production to the stock. (This means that, for example, at the end of the first month they will have a stock of 592 kg.)
- a)** Calculate the total amount of powdered soup sold/given away in 18 months, following the above plan.
- b)** Prove that continuing with the plan over 18 months (theoretically to infinity) will decrease the stock in each consecutive month but it will never go below 250 kg.

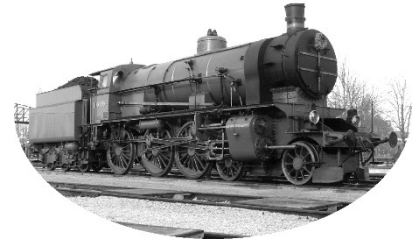
a)	9 points	
b)	7 points	
T.:	16 points	

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9. Steam engine No. 109.109 was built in 1917 and has been preserved in working condition. The diameter of its largest wheel, called the driving wheel, is 1740 mm. The highest permitted speed of the steam engine is 90 km/h.



- a) What is the number of revolutions per minute of the driving wheel when the engine travels at 90 km/h?

The coal consumption of the engine has been investigated on various test drives and it was found that the coal consumption per hour could very well be approximated by the formula $0.5v^2 - 65v + 3800$ kg where v is the average speed in km/h (for $50 < v < 100$). The tender (coal-car) of the steam engine holds a maximum of 6.1 tons of coal.

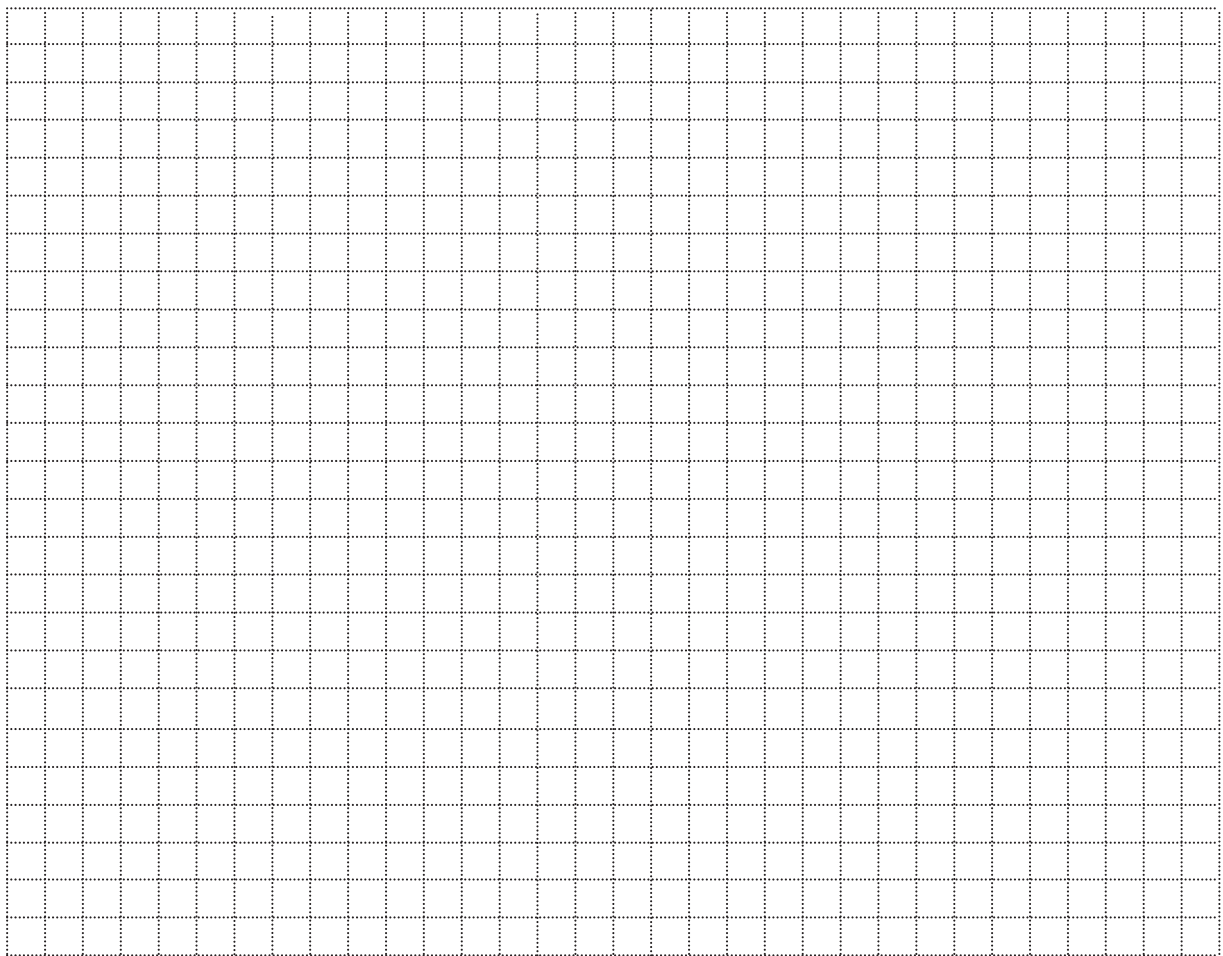
- b) Use the model above to determine how far (in kilometres) the steam engine will travel with 6.1 tons of coal when travelling at an average speed of 60 km/h.
- c) Calculate the average speed at which the 6.1 tons of coal will last for the longest distance.

a)	4 points	
b)	3 points	
c)	9 points	
T.:	16 points	

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	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	12		51	
	2.	13			
	3.	12			
	4.	14			
Part II		16		64	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				115	

date

examiner

	pontszáma egész számra kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző